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ABSTRACT

The purpose of this work is to answer the question if volatility and volatility of volatility are diversifiers, hedges and/or safe havens for the US stock market. Volatility has been an actively discussed topic in the finance community and academia for many decades. The topic of safe havens revived during the Great Recession (2007-2009) and the following Sovereign Debt Crisis in Europe. This thesis will combine these two topics by validating the hedging propositions of VIX- and VVIX-index.

In order to do so, several regression models are conducted. Following Baur and Lucey’s (2010) definitions, this research tested the correlation of each index with the overall stock market. In order to distinguish between times of market turmoil and normal market environment, quantile regressions and Ordinary Least Squares (OLS) models will be utilized. The purpose of the quantile regressions was to prove the safe haven proposition. The OLS models were used in order to identify hedges and diversifiers.

The results show that the VIX is a strong hedging tool for the Standard & Poor's 500 stock index (SPX) with strong negative correlations and high statistical significance. The VIX can also be used as a strong safe haven asset at all times within the observed data set – including several financial crisis. Furthermore, the VVIX can be utilized as a hedge and safe haven for the
SPX - however, correlations are lower and the propositions are weak. Lastly, the VVIX can be a diversifier for VIX futures portfolios. As VIX and VVIX are investable indices, which are traded in real-time, it would be possible to base an actual trading strategy on our findings.

Future research could introduce additional asset classes or equity market indices from outside the United States. The inclusion of other newly issued volatility indices could also be interesting.

Keywords

VIX, VVIX, volatility, volatility of volatility, hedging, diversifier, safe haven, quantile regression, tail risk

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DEDICATION

I would like to dedicate this work to my parents and grandparents whose continued support and encouragement along the way have made this possible.
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I would like to thank my thesis chair Dr. Cetin Ciner for familiarizing me with the topic and his continued support throughout the whole process. My thanks go to my committee for their guidance, and assistance throughout my studies and the thesis.

Furthermore, I would like to thank the staff of Cameron Business School and IGC for assisting us students with diverse problems and questions throughout the overall program.

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SPX  Trademarked Ticker Symbol of S&P 500 Index
US   United States
VAR  Value at Risk
VIX  Trademarked Ticker Symbol by the CBOE Market Volatility Index
VVIX Trademarked Ticker Symbol of VIX of VIX Index; the VVIX is a volatility of volatility measure
\( \beta \) Beta (Greek); market risk\(^1\)
\( \mu \) Mu (Greek); arithmetic mean
\( \sigma \) Sigma (Greek); variance
\( \sigma^2 \) Sigma (Greek) square; standard deviation / volatility

\(^1\) according to ‘Modern Portfolio Theory’ by Markowitz (1952)
INTRODUCTION

Volatility has been an active research topic in the finance community for many decades. The topic of safe havens and tail-risk has also attracted growing interest since the Great Recession (2007-2009) and the consequent Sovereign Debt Crisis in Europe. In recent years, financial markets around the globe have experienced high volatility and unexpected declining returns. Hence investors are keen to protect their portfolios. A key question is which investment vehicles serve as hedges (negatively correlated with stocks) or safe havens (negatively correlated with stocks in extreme stock market declines) in different periods of stock market volatility.

The purpose of this work is to analyze and test the hedging and safe haven proposition of volatility – represented by the VIX – and volatility of volatility – represented by the VVIX – for the US stock market – represented by the S&P 500. In order to do so, several models will be constructed and ordinary least square regressions and quantile regression models performed - Baur and Lucey (2010) or Ciner et al. (2013) used a similar methodology. Hood and Malik (2013) also tested the safe haven proposition of VIX in correlation to other assets including precious metals. However, these papers did not assess volatility of volatility as represented by the VVIX index released by the Chicago Board Options Exchange. The release of this volatility of volatility index has been a leading motivator for this thesis.
Outliers - or often called “tail risk events” - are unlikely to occur, but nevertheless possible. When they occur, the magnitude of these events can be significant. This can lead to great losses in an investor’s portfolio. In order to pursue successful tail risk hedging strategies based on volatility, a certain financial backing, sophistication and a long-term investment horizon are necessary. This thesis particularly addresses the needs of institutional investors such as insurance companies, pension funds and endowments, foundations, hedge funds, asset managers and other financial institutions. Nevertheless, the findings of this paper will also be of interest to retail investors as strategies based on the findings might be possible to pursue by utilizing ETFs, certificates or structured products.

In addition, this paper will also contribute to the ongoing discussion in academia about volatility and hedging tools. The largest contribution will be to the ongoing discussion about the safe haven proposition of the hedge.

The Null-Hypothesis: VIX and VVIX are not affected by (extreme) price movements in equities.

The research question is whether volatility and volatility of volatility are a diversifier, a hedge and/or a safe haven for the US stock market.
Section one introduces and familiarizes the reader with the current academic literature and providing background information on this topic. This section is split into several subchapters and introduces the current understanding of risk and price models. This is followed by an introduction to moments and a review of current research on volatility. Peters (1996) found variance only to be stable and finite for normal distributions. This finding leads to the question whether the dataset under review is under the influence of extreme values or not. As Karunanayake, Valadkhani, and O’Brien (2011) found, a crisis might significantly increase market volatilities.

This leads into an introduction of the two volatility indices in question and their methodologies: VIX and VVIX. Whaley (2009), the creator of the VIX, even suggested the hedging proposition as part of the purpose for creating the VIX. The very strong negative correlation between volatility and stock market prices during a financial crisis could offer a timely protection against financial losses. Guo and Wohar (2006) used the VIX as a mean level of market volatility in order to identify regime shifts in market volatility. As Copeland and Copeland (1999) and Schwert (1989, 1998) observed that aggregated market volatility changes over time, the research data is broken into four arbitrary subsets. Each subset starts with the onset of a financial crisis.
This section is followed by a brief overview of correlation and understanding of heavy/fat tails, as these outlier events are of special interest to us in the context of this work. In addition, the regression methods in this section are explained, which identify the influence of extreme values: quantile regression. The economic approach in this paper is based on a regression model in which volatility indices (VIX and VVIX) are regressed on stock returns and interaction terms that test whether the particular index serves as a hedge or safe haven if the stock market experiences a crisis. A key feature of equity returns is that volatility is time varying and undergoes shifts in variance (see Starica and Granger (2005)).

Section two introduces the utilized data set. Using daily log-returns from January 1990 to May 2013 (for SPX and VIX) and from August 2006 to May 2013 (for VIX, VVIX and SPX) from Bloomberg-Terminal system, several data subsets are created. The cutoff dates for the subsets were selected arbitrary as the onset of a financial crisis. Subsequently the data is split in four subsets: from January 1990 till the beginning of the Asian Financial Crisis in July 1997, from August 1997 to the bursting of the Internet Bubble in March 2000, from April 2000 to the fall of Lehmann Brothers Inc. in September 2008 and from October 2008 to the present, May 2013.
Chapter three introduces the used methodology. This follows Baur and Lucey (2010). The understanding of hedging and safe haven is explained. A hedge, for the purpose of this thesis, is an asset with a negative correlation with the equity market during normal times. It should be noted that this proposition does not need to hold during times of extreme declines in market values; however, if it does, the asset is considered a safe haven as well. From a theoretical perspective, an asset can solely be a hedge, safe haven or both at the same time. If an asset has a positive correlation, it is classified as a diversifier.

Chapter four presents our results and findings, and chapter five contains the resulting conclusions. The results show that VIX serves as a strong hedge with high statistical significance for US stock market and a better safe haven than VVIX during the sample period. The hedge proposition is also stronger than the one of VVIX. This is true during the overall sample period and all subsets, as we can see a lower negative correlation for SPX/VVIX than the correlation for SPX/VIX. As the VVIX has a positive correlation with VIX, the VVIX could be utilized as a diversifier for the VIX as the correlations are positive.

The results suggest that VIX is a superior hedging tool and serves as a better safe haven than VVIX during the sample period. These findings can trigger new hedging and tail risk-hedging strategies for institutional investors. Furthermore, these results confirm Hood and
Malik’s (2013) results and extend them by looking at volatility of volatility as well. The thesis concludes with suggestions for future research.
BACKGROUND & LITERATURE REVIEW

The review of the relevant literature in this paper is divided as follows: first, the literature related to risk and price models are presented, and then secondly moments and volatility as logical parts of price models are also presented. Subsequently, VIX (market volatility index) and VVIX (VIX of VIX index) are assessed, which are indices that measure the volatility and the volatility of the volatility of the Standard & Poor's 500 stock index (S&P500; SPX). Different approaches to coping with time varying volatility in finance are then introduced. This includes exponentially weighted moving average (EWMA), auto-regressive conditional heteroscedasticity (ARCH) and generalized auto-regressive conditional heteroscedasticity (GARCH) models. Thereafter, correlations and hedging as well as their strategic rationale are addressed. The concept of fat tails and safe havens as an extreme type of hedge is also introduced. And finally, this review concludes with the methodology that is used in this paper in order to identify whether the VIX and VVIX are hedges or safe havens.

Risk

Most investors pay close attention to the risk and return proposition of an investment. However, it is much easier to observe returns than risk. Figlewski (1986) introduced several subgroups of risk including credit risk, interest rate risk, and market risk. In general risk can be divided into systematic risks and nonsystematic risks. One typical measure of risk is the variance
of returns or standard deviation – which is a measure describing the squared root mean deviation from the stock or index mean return, also known as volatility. A commonly used ratio in order to measure the risk-return-relation is the Sharpe-Ratio\(^2\) (see Sharpe (1963, 1964 and 1970)). Campbell et al. (2001) showed that idiosyncratic (firm-level) risk grew significantly over the years but that aggregate market volatility has remained stable from 1926 to 1997.

Pricing & Hedging Theory

Amongst many others, Hull (2000) explains the theoretical backgrounds of modern option pricing theory and hedging theory. Most models are still based on a variation of the Black and Scholes’ (1973) option pricing model and variations of the Capital Asset Pricing model (CAPM) (see Black, Jensen and Scholes (1972)). However, the Black and Scholes option pricing model makes certain assumptions (e.g. no time-varying volatility) that, do not match empirical observation of volatility. So-called stochastic volatility models can account for the so called

\[ S = \frac{E[R_a - R_b]}{\sigma} = \frac{E[R_a - R_b]}{\sqrt{\text{var}[R_a - R_b]}}, \]

where \( R_a \) is the asset return, \( R_b \) is the return on a benchmark asset

\(^2\) Since it’s revision in 1994 by the original author William Sharpe, the ex-ante Sharpe Ratio is defined as:
“smile curve” representing the discrepancy between predicted prices by the Black and Scholes model and market-traded option prices.³

Numerous academic articles, such as Merton (1973), explained how option prices are forward-looking. Merton concluded that option writers need to hedge their positions in order to maximize predicted values. Based on their individual pricing models and their individual perception of the market development, traders will either long or short their position(s). Sebehela (2012) focused on exchange options that traded first on an organized exchange in 1973. Most of these studies discuss volatility and present different pricing models. Sebehela assumed that option values are a function of long-term average volatilities, resulting from GARCH call option values.

Hull (2000) went on to explain the models of stock price behavior that are usually described as a Wiener process (a particular type of the Markov stochastic process⁴). According to modern portfolio theory developed by Markowitz (1952), the measure of market risk is a stock’s

⁴ see also Neftci (1996)
Based on Markowitz’ work, the idea of a diversified portfolio drew wide public attention. In this context Samuelson’s (1967) worked on efficient portfolios is noteworthy.

Moments

According to Neftci (1996), moments are one classification used to describe models of distribution functions. Some random variables can be fully characterized by their first two moments such as expected value and mean. Others require higher order moments. The expected value \( E[ X ] \) of a continuous random variable \( X \), with density \( f(x) \), is called the first moment, which is defined by

\[
E[ X ] = xf(x) \, dx,
\]

**Equation 1: Expected value formula**

where \( f(x) \) is the corresponding probability density function.

---

The variance \( \mathbb{E} [ \ X - \mathbb{E}X \ ]^2 \) is the second central moment around the mean. The first moment of a random variable is the “center of gravity” of the distribution, while the second moment gives us information about the way the distribution is disbursed. If the distribution is not centered around the mean, higher order moments describe this behavior. For example, skewness (third moment) or kurtosis (forth moment) provide information about such asymmetries. However, for this thesis the phenomenon of heavy tails is the important notion. Fourth moments are used for this purpose.

Volatility

The variance is the expected value of the square of the difference between a particular outcome and its mean (or the root mean squared deviation of the difference between a particular outcome and its mean):

\[
\sigma^2 = \text{Var} [ \ X \ ] = \mathbb{E} [ ( X - \mu)^2 ],
\]

**Equation 2:** Variance of Returns

where:

\( \sigma^2 \) or \( \text{Var} [ \ X \ ] \) represents the variance of the uncertain return \( X \)

\( \mu \) its mean
and E [.] denotes the arithmetic mean or expected value of the expression in brackets

The standard deviation is more commonly used in finance since it is expressed as the same units as the mean. Peters (1996) reminded us that variance is only stable and finite for normal distributions. Such instances are rare in financial markets and one may not be able to accept a “stable paretian” family of distribution as postulated by Mandelbrot (1960, 1961 and 1963).6

Volatility is the most important measure of risk in asset prices. This supported illustrating that simple models of market efficiency are prone to error. Le Roy and Porter (1981) and Shiller (1979) published initial studies on this topic. Natenberg (1988) delivered a condensed overview of option volatility and pricing strategies. Most option valuation models consider volatility two-dimensional: historic and implied (see e.g. Koziol (1990), Hull (2000) or Dumas, Fleming and Whaley (1998)). In trading, the expected value is considered more important than the fair value.

There are numerous studies on the causes of volatility reaching as far back as to the 1960s. Some of the most prominent research results come from Fama (1965), French (1980),

6 See Pareto, V. (1897), Cours d’Economie Politique, Lausanne, Switzerland
French and Roll (1986) and Roll (1984). More recent studies by Karunanayake, Valadkhani, and O’Brien (2010) focused on volatility transmission amongst international markets. Based on their results, there are no significant returns arising from a crisis (Karunanayake, Valadkhani, and O’Brien (2010)). However, a crisis might significantly increase market volatilities, larger markets seem to have a greater influence on other markets. The authors conclude that diversification amongst international markets might not protect an investor from a crisis event due to a great level of time-varying co-volatility. Further noteworthy studies came from Kim and Rogers (1995), Chan et al (1997), Kanas (1998) Chou et al. (1999), Reyes (2001), Hassan and Malik (2007) and Harju and Hussain (2008). These contributed significant developments to the literature at hand. Ciner and Sackley (2007) tested the often mentioned positive relationship between volatility and volume for the emerging market of Taiwan by means of daily transaction volume. They found that the volume-volatility relation is entirely driven by the daily number of trades. Similar work has been published by Clark (1973) and Jones et al. (1994).

Relevant research in regard to market liberalization, volatility and outliers has been conducted by Eizaguirre et al. (2009). The authors find that the liberalization of financial markets in emerging economies generates outlying returns around the opening dates. These outliers may appear to mimic an increase in volatility. After correcting for this effect, the authors found that
volatility in Latin American markets seems to decrease after the liberalization of financial markets, but Asian markets seem to suffer from increased volatility. It is important to note, that all markets seem to suffer more often than not from sudden significant shocks. Furthermore, large financial (crisis) events seem to have a significant impact on a global level. Similar research has been performed by De Santis and Imrohoroglu (1997), Huang and Yang (1999), Kim and Singal (2000), Kaminsky and Schmukler (2003), and Jayasuriya (2005). Research on the same topic for mature markets has been performed by Franses and Ghijsels (1999), Johansen and Sornette (2001), and Charles and Darne (2005). These articles conclude that outlier returns stem from the data generating process.

Another branch of research focuses on regime shifts in market volatility. Guo and Wohar (2006) used the VIX as a mean level of market volatility. Copeland and Copeland (1999) and Schwert (1989, 1998) also observed that aggregated market volatility changes over time.

Many studies suggest that equity market volatility is asymmetric. This means returns and conditional volatility are negatively correlated. Christie (1982) explained the asymmetry based on the “leverage hypothesis”. These “leverage effects” have become a synonym for asymmetric volatility. The asymmetric results can be created by a volatility feedback driven by a time varying risk premium as documented by Campbell and Hentschel (1992). They conclude that
news brings higher volatility. This volatility leads to a higher required return, which results in a stock price decline. Current research indicates that both of these reasons do not completely explain the asymmetry, the two effects are in play simultaneously, as shown by Bekaert and Wu (2000). The negative relation seems to be stronger than expected, according to Malik (2011).

Several studies focus on variance swaps as a hedging tool for volatility. Since volatility is the square root of variance, academics (as Broadie and Jain (2008)) and market practitioners consider variance swaps to be the “purer” product exposing the investor to price variation. Broadie and Jain (2008) studied hedging and pricing of variance swaps.

VIX & VVIX

In 1993, the Chicago Board Options Exchange (CBOE) released a volatility index called VIX (trademarked ticker symbol by the CBOE Market Volatility index). On September 22nd, 2003, the CBOE updated the definition and calculation of the VIX index. Further back-calculated data based on option prices till 1990 have been included.\(^8\) Whaley (1993, and 2000) coined the notion of “Investor fear gauge” because the VIX captures the expected future volatility for the upcoming 30 days based on implied volatility of at-the-money options on the

\(^8\) see [http://www.cboe.com/micro/VIX/vixintro.aspx](http://www.cboe.com/micro/VIX/vixintro.aspx) for more details
Standard & Poor’s 100 (S&P100; OEX). In addition to the new methodology, the VIX started to be based on SPX instead of OEX⁹. Jiang and Tian (2007) examined the new VIX calculation procedure following the CBOE redesigned the methodology. They found that the present methodology might underestimate true volatility by as much as 1.98% and suggest easy solutions to the problem using simple smoothing methods.

The S&P 500 VIX Futures Index Series seeks to model the outcome of holding long and/or short positions in VIX futures contract or other VIX indices. Historically, the VIX index has a negative correlation to the SPX and is considered a useful hedging tool of the broad equity market. While the spot VIX is difficult to replicate as a practical matter, there is a market in VIX futures and options. There are also several sub-indices available.

The indices model returns from long VIX futures position that is rolled continuously throughout the period between futures expiration dates. The VIX is computed on a real-time basis throughout each trading day and is of a forward-looking nature. Conceptually, the VIX is, as Whaley (2009) explained, like a bond’s yield to maturity. As the VVIX is using the same calculation method based on VIX options and VIX futures contracts instead of SPX options and

---

⁹ see Carr and Wu (2006) for a comparison of the two methodologies
SPX futures contracts. The introduction to the calculation methodology on the VVIX is discussed in the following section.\(^\text{10}\)

The VVIX is a fairly new index\(^\text{11}\) that is a volatility of volatility measure in that it represents the expected volatility of the 30-day forward price of the VIX nearby options. It is calculated from the price of a basket of liquid at- and out-of-the-money VIX options. The methodology used is the same as for the VIX. Prices for the VVIX are posted in real time and its term structure is posted on CBOE’s website.\(^\text{12}\) VVIX and its term structure convey:

- The expected volatility that determines VIX option prices.
- The expected volatility of the VIX itself to a nearby horizon.
- The mean and standard distribution of settlement values of VIX futures and options.

Different points on the VVIX term structure are pricing portfolios of VIX options (VVIX portfolio) to different expirations. A position in a VVIX portfolio replicates the volatility of VIX


\(^{11}\) Publication started March 14, 2012

forward prices. VVIX prices have usually been at a premium relative to future realized volatility.

The discount is a volatility risk premium as illustrated in Figure 1.

![Figure 1: Time series of the VVIX and VIX between June 2006 and February 2012](image)

The range of values of the VVIX is at a significantly higher level than that of the VIX. It ranges between 60 and 145 around an average of 86. The VIX ranges between 10 and 81 around an average of 24. Note that the range of variation of the VVIX tends to widen at higher values of the VIX.

Except at high values of the VIX, there is little correlation between variations of the VIX and the VVIX. Furthermore, the VVIX tends to revert to its historical mean.
The VVIX exhibits a downward sloping term structure, as illustrated in Figure 2.

Figure 2: Term Structure of VVIX

Points on the VVIX term structure yield estimates of the fair value of the VIX at the expiration dates. Those points on the VVIX term structure also proxy for the standard deviation of the VIX at expiration dates.\textsuperscript{13}

\textsuperscript{13} See CBOE (2012b) for more details
The VVIX is calculated with the same methodology as VIX. At each expiration, VVIX is calculated from VIX options prices using the VIX formula\textsuperscript{14} from equation 3:

\[
\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[ \frac{F}{K_0} - 1 \right]^2,
\]

where...

- $\sigma$ is \(\frac{VVIX}{100} \Rightarrow VVIX = \sigma \times 100$
- $T$ Time to expiration
- $F$ Forward index level derived from index option prices
- $K_0$ First strike below the forward index level, F
- $K_i$ Strike price of \(i\)\textsuperscript{th} out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i < K_0$; both put and call if $K_i = K_0$.
- $\Delta K_i$ Interval between strike prices – half the difference between the strike on either side of $K_i$:

\[
\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}
\]

(Note: $\Delta K$ for the lowest strike is simply the difference between the lowest strike and the next higher strike.

\textsuperscript{14} Please find more details for the VIX methodology under http://www.cboe.com/micro/VIX/VIXwhite.pdf
Equation 3: VIX / VVIX methodology\textsuperscript{15}

Some studies suggest informational content of the VIX index that allows market timing for switching between two portfolios (see Copeland and Copeland, (1999)). Hyerczyk (2001) found that utilizing the VIX may enhance the performance of trading signals. Corrado and Miller (2005) investigated if the volatility index has predictive powers.

Hood and Malik (2013) conducted recent research on investigating whether gold can act as a hedge or safe haven for stocks. The authors also looked at the VIX as hedge and safe haven for the US-stock market.\textsuperscript{16} Hood and Malik used the Iterative Cumulative Sums of Squares (ICSS) algorithm, developed by Inclan and Tiao (1994). This endogenous method is used to search for structural breaks or shifts in variance. They find the VIX a very successful hedge and safe haven. Among others, Aggarwal, Inclan, and Leal (1999) and Ewing and Malik (2005) have also used this methodology.

Hood and Malik (2013) found a strong significant negative correlation between volatility and stocks during adverse market movements, offering great protection potential for investors.

\textsuperscript{15} From \url{http://www.cboe.com/micro/VIX/VIXwhite.pdf} ; retrieved 09/23/2013, 12:30 PM
\textsuperscript{16} The US-stock market is the biggest financial market in the world, followed by Japan and UK. (Standard and Poor’s (2008)).
Hilal, Poon and Tawn (2011) discussed the optimal hedge ratio to be used in order to hedge for extreme values with the VIX.

The findings of Ang, Bekaert, and Liu (2005) are also interesting in this context. They argued that investors rapidly switch between assets. This expresses that extreme price movements can have informative value, which further motivated the consideration of the VVIX as well for this study.

EWMA, ARCH & GARCH

The following section introduces commonly used methodologies and models. The GARCH model proposed by Bollerslev (1986) seems to be a widely followed approached in academia. The difference between the EWMA model and GARCH is that $\sigma^2$ is calculated from a long-run average variance rate, $V_L$, as well as from $\sigma_{n-1}$ and $u_{n-1}$. The equation for GARCH is

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$
Equation 4: GARCH equation following Bollerslev (1986)

where:

\[ \gamma \]
\[\text{is the weight assigned to } V_L\]

\[ \alpha \]
\[\text{is the weight assigned to } u^2_{n-1}\]

\[ \beta \]
\[\text{is the weight assigned to } \sigma^2_{n-1}\]

\[ u_i = \ln \left( \frac{S_i}{S_{i-1}} \right) \]

and \( S_i \) is the value of variable at the end of the \( i \)-th, for \( i = 0,1, \ldots, n \)

The EWMA assigns a higher weight to recent observations. GARCH, on the other hand can also be used for updating covariance rate estimates and forecasting of future covariance rates.

Hedging ideas that go beyond Black-Scholes delta hedging are presented in Takahashi et al. (2011) and involve polynomial variance swaps. Hull (2007) went on to explain the basic concepts of EWMA, ARCH and generalized GARCH. The distinctive feature of these models is the implementation of volatility as a non-constant value and the tracking of variations in volatility through time.
Engle’s (1982, 1993 and 1995) work on GARCH delivered a significant contribution to the recent literature. His most recent work on the topic (2008) focused on low frequency volatility. Other works such as Nelson (1990), Noh, Engle and Kane (1994) and Vasicek (1987) focused on the influence on asset returns and option prices.

The correlation of a diversifier is only required to hold most of the time – not always – according to Baur and Lucey’s (2010) definition. Amongst others, Ciner et al. (2013) agreed with this definition. Noteworthy, in the context of this thesis, is Szado (2009), because the author identified the VIX as a diversifier and a potential portfolio protection in downturns. Szado reasons that this diversification preposition could be very helpful in a crisis when many asset classes tend to show increased correlations. Szado did not use quantile regressions and did also not include volatility of volatility.

Correlations

The correlation coefficient, $p$, between two variables $V_1$ and $V_2$ is defined as:

$$p = \frac{[ E (V_1 V_2) - E (V_1) E (V_2) ]}{[ SD (V_1) SD (V_2) ]}$$

**Equation 5:** Correlation coefficient

where:
E(·) denotes expected value
SD(·) denotes standard deviation
and \( E(V_1 V_2) = E(V_1) E(V_2) \)

The covariance between \( V_1 \) and \( V_2 \) is defined as

\[
\text{cov}(V_1, V_2) = E(V_1 V_2) - E(V_1) E(V_2)
\]

**Equation 6:** Covariance between \( V_1 \) and \( V_2 \)

so that the correlation can be written as:

\[
p = \frac{\text{cov}(V_1, V_2)}{\text{SD}(V_1) \text{SD}(V_2)}
\]

**Equation 7:** Correlation rewritten

Hull (2007) observed covariances as fundamental variables of EWMA and GARCH. As mentioned above, many studies focus on financial market correlations. E.g. Savva (2009) found volatility spillovers between the US and European markets as well as reverse spillovers. Furthermore, the intensity of correlation amongst markets is higher, not only for negative shocks, but also when several shocks with opposite signs occur. It has been generally accepted that multivariate models are appropriate for transmission mechanisms and correlation dynamics (see Martens and Poon (2001)). This behavior is also known under the term “co-movement”.

25
Dajcman (2012) presented a recent study on co-movement of German bonds, which the public regards as safe haven, and equity markets. The research exposes a nearly positive relation between German stocks and German bonds, however, the correlation is mostly negative for countries affected by the European credit crisis. A long-term study, covering 150 years, was performed by Goetzmann, Li, and Rouwenhorst (2005). The authors found that correlations amongst markets were strongly influenced by globalization of markets. Their findings suggest that better diversification can be achieved by utilizing less explored markets. Hood and Malik (2013) reasoned, that the relatively unexplored VIX market might provide a better hedge and safe haven opportunity for investors. Also, Sing, Kumar and Pandey (2010) examined volatility spillovers, however, the authors used Value at Risk (VAR) as a determining variable in their model.

Using a comparison from medicine regarding how infectious diseases spread, the phenomenon of increased correlation of financial markets during a market crash is often referred to as “contagion”. Bekaert, Harvey and Ng (2005) noted that there seems to be no precise definition of the use of the term in finance. Forbes and Ribobon (2001) stated:
“There is no consensus on exactly what constitutes contagion or how it should be defined.”

Rigobon (2002) wrote:

“Paradoxically, …there is no accordance on what contagion means.”

Bekaert, Harvey and Ng showed in their research that correlation is a key factor in a contagious market environment.

The proper calculation of dynamic correlation becomes crucial in the context of estimation process of hedge ratios. If volatilities and correlations are changing, then hedge ratios should be adjusted accordingly (see Engle (2002)).

Hedging

Kramer, Miller-Sennholz and Helstrom (1985) described index options as options on current or “spot” indices that are settled in cash upon exercise. The underlying asset for a futures option is a futures contract. Exercising of such an option would then lead to a long or short position in the underlying futures contract. Koziol (1990) explained the hedging process itself as
a multivariate risk management process which can achieve multiple functions as portfolio insurance, improvement of capital management or pricing of offerings. Hull (2007) also discussed the timing of when to hedge and in which competitive environment hedging makes sense. In his work Hull also explained the basic concept of delta and gamma-hedging and other “Greeks”.

Heavy / Fat Tails

If a distribution has heavier or fat tails than a normal curve, it has a higher probability of extreme observations (see Neftci, (1996)). While a normal distribution can also take extreme values, in the case of a heavy-tailed distribution, these observations have a higher frequency. Furthermore, the middle tail region of the distribution contains relatively less weight than the normal density. Excess kurtosis and volatility clustering are common for financial data. Extreme observations can have a significant influence on model results (Franses, Van Dijk and Lucas (2004)). This type of distribution is challenging to model with GARCH.

17 See Fouque, Jean-Pierre, Papanicolaou, George, and Sircar, Ronnie K. (2000) for further details on Black-Scholes Delta Hedging

18 see Kyrtou and Terraza (2008) and therefore it is suggested to conduct a Mackey-Glass-GARCH in order obtain more accurate results.
The multivariate form of extreme value theory is difficult to measure in financial markets due to prevalent heteroscedasticity in financial time series. Many approaches are based on a limiting argument in which all variables become large at the same time. Hilal, Poon and Tawn (2011) demonstrated how the conditional approach of Heffernan and Tawn (2004) could be used to model external dependence between financial time series.

Taleb (2007) labeled these fat-tail events “black swans”, which he describes as highly consequential but unlikely events. He believes that scientists rely on standard statistical tools such as the bell curve too much and recommends “power-law-distribution”.

Furthermore, Bhansali and Davis (2010) and Bhansali (2008) showed that tail risk hedging could not only protect a portfolio against these extreme adverse events, but also additionally diversify a portfolio and create profits on an ongoing basis.

Safe Haven

Testing financial market linkages under extreme adverse environments is related to the field of behavioral finance and the prospect theory as reactions of market participants might be steered by emotions only. Kahneman and Tversky (1979) realized that investors react more sensitively to losses than they do to gains. These findings are also consistent with more recent studies. Loss-averse behavior of humans has been confirmed by Brooks and Zank (2005).
Interesting is their discovery of a gender effect: women are proportionally more loss averse. The findings of Ang, Bekaert, and Liu (2005) are also interesting in this context. They argue that investors switch between assets rapidly. This means that extreme price movements can have informative value. This motivated this research to consider the VVIX as well for this study.

Linkages between financial prices during times of crisis have been the subject of many studies that try to determine if tail-events occur at the same time in different markets. Recent works include Hartmann, Straetmans, and de Vries (2004) who investigate connections between equity and fixed income markets. On the other hand Cumperayot, Keijzer, and Kouwenberg (2006), examined linkages between equity market and foreign exchange returns in periods of a market shock.

Other studies include liquidity measures, such as bid-ask-spreads, besides volatility of prices, in order to establish the safe haven preposition of certain assets (see Upper, Deutsche Bundesbank, (2000)). In the context of tail risk, Hart and Tauman, (2004) examined reasons for market crashes. Using historical data, the authors observed, that a market crash does not need to be followed by a recession. Moreover, they noticed that in the days preceding such a market crash, there generally were, no significant external events to justify the drop, which they labeled
“bad news”. Hart and Tauman argued that these crashes might have been the result of information processing of market participants.

A safe haven study on foreign exchange was performed by Ranaldo and Söderlind (2010). Their work focused on the linkage between increased volatility levels in stock markets and investors flight to the Swiss Franc and Japanese Yen.

Quantile Regression

Ordinary least-squares regression models are widely used. The methodology models one or more covariates $X$ and the conditional mean of a response variable $Y$ given $X = x$. Quantile regression, on the other hand, models the relationship between $X$ and the conditional quantiles of $Y$ given $X = x$. This proposition makes quantile regression very helpful in cases where extreme values are important (see Chen and Wei (2005)).

Koenker (2005) established down the basic principles of quantile regression that are then applied by Flom (2011). Ordinary least squares (OLS) regression models the conditional value of the dependent variable as a function of one or more independent variables. Since the mean is not a full description of a distribution, modeling the mean is not a full description of a relationship between dependent and independent variables either. Quantile regression models the conditional
quantiles rather than the mean. This is of great advantage to highly skewed distributions as for example income, or financial data.

A quantile is an order statistic. The p-th (samples)/(population) percentile is the value that is higher than p% of all the values in the (sample)/(population). The t-th quantile of X is defined as

\[ F^{-1}(t) = \inf \{ x : F(x) > t \}, \]

**Equation 9:** t-th quantile of X

where F is the distribution of X

As noted by Fox and Rubin (1964), Koenker and Hallock (2001) and Koenker (2005), the problem of sorting can be converted into one of optimization. The problem is to minimize:

\[ E \rho_t (X - \hat{x}) = (\tau - 1) \int_{-\infty}^{\hat{x}} (x - \hat{x}) dF(x) + \tau \int_{\hat{x}}^{\infty} (x - \hat{x}) dF(x) \]

**Equation 10:** Quantile regression

This allows a simple extension of the problem of ordinary least squares regression to quantile regression. Mosteller and Turkey (1977) remarked in their work of least squares:
What the regression curve does is give a grand summary for the averages of the distributions corresponding to the set of x’s. We could go further and compute several different regression curves corresponding to the various percentage points of the distributions and thus get a more complete picture of the set. Ordinarily this is not done, and so regression often gives a rather incomplete picture. Just as the mean gives an incomplete picture of a single distribution, so the regression curve gives a corresponding incomplete picture for a set of distributions.

Koenker noted, that quantile regression could help to develop a more comprehensive picture of overall regressions. Xiao and Koernker (2009) showed how to use quantile regression methods and expressed how they can outperform some existing conditional quantile estimation models. Quantile regression estimation of GARCH models is highly nonlinear. The authors proposed a two-step approach in order to estimate linear GARCH time series.

Prior studies as e.g. Baur and Lucey (2010) or Ciner et al. (2013), suggested that “extreme” events could influence the distribution of financial returns beyond regular fluctuations. In this study, the focus remains on the VIX and VVIX and their ability to provide protection during extreme adverse movements of the S&P 500. A discussion of the specific
properties of each volatility index follows in the next chapter. Prior work focuses on conventional asset classes as equities, bonds, commodities or foreign exchange.
DATA

The data consist of daily closing spot prices for the VIX index, VVIX index, and the S&P 500 Index. All the data used in the paper was obtained from a Bloomberg-Terminal. The data covers the period from January 2, 1990 to May 31, 2013. Our sample period is particularly interesting since it includes several financial crisis events the Asian Crisis of 1997, the burst of the so called Internet Bubble 2001 and the most recent Financial Crisis of 2008-09. All prices are quoted in US-Dollars and all indices’ closing values occur at 4 p.m. of every trading day. The sample includes 5,893 observations for the SPX and VIX. The data used for the VVIX start on August 21, 2006 and the set includes 1,701 observations. Log-returns have been used for all analysis conducted in this study.
Table 1: Descriptive Statistics VIX, VVIX and SPX

<table>
<thead>
<tr>
<th></th>
<th>VIX</th>
<th>VVIX</th>
<th>SPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.03%</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.08%</td>
<td>0.12%</td>
<td>0.02%</td>
</tr>
<tr>
<td>Median</td>
<td>-0.31%</td>
<td>-0.43%</td>
<td>0.05%</td>
</tr>
<tr>
<td>Mode</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.06118</td>
<td>0.05152</td>
<td>0.01166</td>
</tr>
<tr>
<td>Sample Variance</td>
<td>0.00374</td>
<td>0.00265</td>
<td>0.00014</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.38949</td>
<td>18.3927</td>
<td>8.5226</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.66969</td>
<td>0.85242</td>
<td>-0.23219</td>
</tr>
<tr>
<td>Range</td>
<td>0.8466</td>
<td>1.04415</td>
<td>0.20427</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.35059</td>
<td>-0.52446</td>
<td>-0.0947</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.49601</td>
<td>0.5197</td>
<td>0.10957</td>
</tr>
<tr>
<td>Count</td>
<td>5893</td>
<td>1701</td>
<td>5893</td>
</tr>
</tbody>
</table>

Table 1 provides descriptive statistics for all three series under study. The VVIX has the highest arithmetic mean and the VIX has the lowest arithmetic mean of the three series. However, this is not surprising due to the mean reverting nature of volatility. The standard deviation of the VIX is more than five times greater than the one of the SPX and also the highest of the three series. All three data sets – and especially the VVIX – show high values of kurtosis.
The correlation between the VIX and SPX is -0.7083 during the overall period. The correlations for VIX, VVIX and SPX for the time since inception of the VVIX up to the bankruptcy of Lehmann Brothers Inc. and from this time event to May 2013 are shown in table 2 and table 3. From these tables an increase of the negative correlation between the VIX and the SPX and the VVIX and the SPX becomes determined. This negative correlation between SPX and VVIX almost doubled. Furthermore, it is noticeable that the correlation between the VIX and VVIX nearly doubled after the Lehmann crisis.

<table>
<thead>
<tr>
<th>VIX</th>
<th>SPX</th>
<th>VVIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>SPX</td>
<td>-0.751199</td>
<td>1</td>
</tr>
<tr>
<td>VVIX</td>
<td>0.4769223</td>
<td>-0.341697</td>
</tr>
</tbody>
</table>

Table 2: Correlation of VIX, VVIX and SPX until Lehmann Crisis

<table>
<thead>
<tr>
<th>VIX</th>
<th>SPX</th>
<th>VVIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>SPX</td>
<td>-0.824961</td>
<td>1</td>
</tr>
<tr>
<td>VVIX</td>
<td>0.8037709</td>
<td>-0.6602805</td>
</tr>
</tbody>
</table>

Table 3: Correlation of VIX, VVIX and SPX after the Lehmann Crisis
The overall data set has been divided into four subsets in order to capture the different market behavior of the three series under analysis and the influence of different market regimes. The cut-off dates have been set some sort of arbitrary to the beginning of each crisis. The first subset starts on January 1, 1990 and ends on July 31, 1997 with the onset of the Asian crisis. The second subset consequently starts on August 31, 1997 and ends March 31, 2000 with the bursting of the so-called Internet Bubble. The third subset begins with April 3, 2000 and ends on September 30, 2009. This period includes the inception of the VVIX. We included this data starting from August 21, 2006. The fourth and last subset begins with the Lehmann Brothers bankruptcy in September 2009 and ends on May 31, 2013. An overview of descriptive statistics of all subsets compared to the overall series is provided in table 4. It might not surprise to see the lowest SPX returns occurring in subset 4 from inception of VVIX until the Financial Crisis. Subsequently these negative returns of the SPX occurred with the highest standard deviation for the SPX recorded in the study’s data. The log-returns of the SPX showed the highest kurtosis in subset 3 and from inception of the VVIX till Lehmann crisis. All subsets of the SPX exhibit negative skewness except subset 3 and the set from inception of the VVIX till Lehmann crisis. It is worth to note that the mean returns for VVIX are the highest of the three series discussed in all subsets during which the VVIX existed. The VIX index exhibits the highest standard deviation in all subsets. Table 4 provides a full overview of the correlations of each subset. The data suggest
that the correlation between VIX and SPX has been lowest before the Asian Crisis and highest during the period between Asian crisis and Internet Bubble. The values for correlation before after the Credit Crisis on the other hand are closer to the correlation of the overall data set.
Table 4: Overview of correlation matrixes

<table>
<thead>
<tr>
<th>VIX/SPX</th>
<th>SPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>1</td>
</tr>
<tr>
<td>SPX</td>
<td>-0.7083</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VIX/SPX before Asian Crisis</th>
<th>SPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>1</td>
</tr>
<tr>
<td>SPX</td>
<td>-0.56728</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VIX/SPX after Asian crisis</th>
<th>SPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>1</td>
</tr>
<tr>
<td>SPX</td>
<td>-0.8015</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VIX/SPX after Internet crisis</th>
<th>SPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>1</td>
</tr>
<tr>
<td>SPX</td>
<td>-0.7433</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VIX/SPX/VVIX since VVIX (after Internet till Lehmann)</th>
<th>VVIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>1</td>
</tr>
<tr>
<td>SPX</td>
<td>-0.7512</td>
</tr>
<tr>
<td>VVIX</td>
<td>0.476922</td>
</tr>
<tr>
<td></td>
<td>A1-MNV</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>Mult</td>
<td>0.00%</td>
</tr>
<tr>
<td>Min</td>
<td>-0.31%</td>
</tr>
<tr>
<td>Max</td>
<td>6.12%</td>
</tr>
<tr>
<td>Mean</td>
<td>4.32%</td>
</tr>
<tr>
<td>Std</td>
<td>0.67%</td>
</tr>
<tr>
<td>Min</td>
<td>-35.66%</td>
</tr>
<tr>
<td>Max</td>
<td>43.60%</td>
</tr>
<tr>
<td>Mean</td>
<td>6359</td>
</tr>
</tbody>
</table>

Table 5: Overview - Descriptive Statistics of Subsets.
METHODOLOGY

In the following section we will discuss the methodology used for this thesis in order to analyze the safe heaven and hedge property of the two volatility indices relative to the overall stock market, represented by the SPX. The used econometric model follows Baur and Lucey (2010):

\[ R_{VIX, VVIX_t} = a + \beta_{stock_t} + \beta_{stock_t(q)} + \varepsilon \]

**Equation 8**: Regression model

where:

\( \beta_{stock_t} = \) stock returns during normal times

\( \beta_{stock_t(q)} = \) stock returns during crisis times

\( \varepsilon = \) error term

Equation 8 models the relationship between the volatility indices and the stock returns. We assume that relationships are not constant but influenced by extreme market conditions. The error term of our model is given by \( \varepsilon \). If \( \beta_{stock_t} \) is significantly different from zero, then there is evidence of a relationship between the index and the equity market. If \( \beta_{stock_t(q)} \) is negative and statistically different from zero, the index can work as a strong safe haven. In case the parameters
are non-positive, then the asset would be a weak safe haven. Bekaert, Harvey and Ng (2005) showed in their paper that correlation is a key factor in a contagious market environment.

The difference between this paper’s work and Hood and Malik (2013) is the methodology used and the extension of this paper’s research by the utilization of the volatility of volatility index VVIX and quantile regression. Taleb (2007) noted out that hedges against negative black swans are possible whilst having the chance to benefit from positive ones. The illustration of this effect is attempted in this paper by using the VIX / VVIX as hedge / safe haven asset.

Hedging

If investors can add an asset to their portfolio that reduces losses in times of crises (so called “heavy-tail-events” or “tail risk”) by more than hedge or diversifier assets they will be able to reduce losses to their portfolios and increase overall market stability. In order to distinguish a hedge and a diversifier, Baur and Lucey’s (2010) defined a hedge asset, a diversifier asset and a safe haven asset in this work.

They defined a hedge as follows:
“A hedge is defined as an asset that is uncorrelated or negatively correlated with another asset or portfolio on average.”

A hedge asset does not protect against losses in times of turmoil per se. In periods that are dominated by panic, asset classes tend to experience different correlation relationships than during normal times. Overall, recent studies found an increased multi-asset-correlation during crisis times.

Baur and Lucey defined a diversifier as:

“A diversifier is defined as an asset that is positively (but not perfectly correlated) with another asset or portfolio on average.”

This definition is close to the definition of a hedge since the loss-reduction property is not required to hold in extreme adverse market environments. The correlation of the diversifier is only required to hold most of the time – not always.
Safe Haven

Baur and Lucey (2010) defined a safe haven as below – we will follow their definition in this piece of work.

“A safe haven is defined as an asset that is uncorrelated or negatively correlated with another asset or portfolio in times of market stress or turmoil.”

The distinguishing property in this definition is the negative correlation in crisis times. This property does not force the correlation to be positive or negative on average but only to be zero or negative during a crisis. On average, the correlation can be positive or negative. A negative correlation during times of adverse market movements will compensate an investor for losses in her portfolio. Baur and Lucey’s definition of a safe haven was consistent with that from Webster’s dictionary.\(^{19}\) In fact, Baur and Lucey’s definition seemed to match Kaul and Sapp’s (2006) definition and has also been adopted by Joy (2011).

In order to test for safe haven properties in this paper, Ciner et al. (2013) and Baur and Lucey’s (2010) approach was followed. Both studies follow a quantile regression approach in

order to test correlations in case of times of crisis. Ciner et al. and Bauer and Lucey focus on moves in the lower five quantiles. This is equivalent to a standard deviation of two \( \sigma \).

Bae, Karolyi, and Stulz (2003), Baur and Lucey (2010) and Cetin et al (2013) utilized quantile regressions in their work and included regressors that contain returns that are in the \( q\% \) quantile, such as the 5%, 2.5% and 1% quantile. This paper’s research followed their choice - which is arbitrary to a certain degree.

The Null-Hypothesis: VIX and VVIX are not affected by (extreme) price movements in equities.

The research question is whether volatility and volatility of volatility are a diversifier, a hedge and/or a safe haven.
RESULTS

This section presents the results from the model estimated above. All regressions have been run at the 95% confidence level. Models of OLS regression for all data sets and subsets of SPX/VIX and SPX/VVIX were conducted. The purpose of the OLS models was to present an indication about the hedge and diversifier proposition of the volatility indices. These models are covered normal times.

In order to test the safe haven proposition, a quantile regression models was used to illustrate if there is a different behavior of the volatility indices in times of extreme stock market declines. If the regression models of the quantile regression presented significantly different results, this would mean that outliers / extreme values have a significant influence on the distribution.
Table 6: Overview of OLS-Regression Models

<table>
<thead>
<tr>
<th></th>
<th>Subet 1</th>
<th>Subet 2</th>
<th>Subet 3</th>
<th>Subet 4</th>
<th>Subet 5</th>
<th>Subet 6</th>
<th>Subet 7</th>
<th>Subet 8</th>
<th>Subet 9</th>
<th>Subet 10</th>
<th>Subet 11</th>
<th>Subet 12</th>
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<td>R²</td>
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<td>0.3233</td>
<td>0.6431</td>
<td>0.5777</td>
<td>0.6118</td>
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<td>Adjusted R²</td>
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<td>0.3771</td>
<td>0.4771</td>
<td>0.4271</td>
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<td>0.3861</td>
<td>0.4261</td>
<td>0.2611</td>
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<td>Standard Error</td>
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<td>2.3202</td>
<td>2.3202</td>
<td>2.3202</td>
<td>2.3202</td>
<td>2.3202</td>
<td>2.3202</td>
<td>2.3202</td>
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<tr>
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<td>0.0963</td>
<td>0.0963</td>
<td>0.0963</td>
<td>0.0963</td>
<td>0.0963</td>
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<tr>
<td>X1 Variable 4</td>
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</table>
Table 6 shows the estimates of the OLS regression model given by equation 10. This represents the market during normal times and allows testing the hedge and diversifier preposition. Five regressions of VIX on SPX were conducted, three regressions of VVIX on SPX and three regressions of VVIX on VIX for the OLS analysis.

The p-values of all VIX-regressions indicate statistical high significance of the regression models, followed by high $R^2$-values. The p-values of all VVIX-regression models indicate statistical significance, however the VVIX regressions on the VIX have little statistical significance.

Observing at the correlations between the different datasets show a high negative correlation for the VIX and SPX for all subsets. This indicates a strong hedge proposition of the VIX for the SPX - following our definitions - with high statistical significance.

The correlation coefficients of VVIX and SPX have only small negative values and low $R^2$-values. Additionally, the VVIX can be used as a hedge for the SPX however, the hedge proposition is smaller and the statistical significance is lower compared to the VIX.

Furthermore, VVIX and VIX have a positive correlation coefficient however, they also have low $R^2$-values. Following the aforementioned definitions, the VVIX cannot be used as a hedge for the VVIX. Nevertheless, the VVIX co-moves with the VIX and can be used as a VIX-diversifier.
### Table 7: Overview all Quantile-Regression Models

<table>
<thead>
<tr>
<th>Observations</th>
<th>WVS</th>
<th>PK</th>
<th>WVS</th>
<th>PK</th>
<th>WVS</th>
<th>PK</th>
<th>WVS</th>
<th>PK</th>
<th>WVS</th>
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<th>WVS</th>
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<tbody>
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<td>5%</td>
<td>5%</td>
<td>5%</td>
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<td>5%</td>
<td>5%</td>
<td>5%</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
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<td>0.0005</td>
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<td>-0.129</td>
<td>-0.129</td>
<td>-0.129</td>
<td>-0.129</td>
<td>-0.129</td>
<td>-0.129</td>
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<td>-0.129</td>
<td>-0.129</td>
<td>-0.129</td>
<td></td>
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<tr>
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<tr>
<td>Intercept</td>
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<td>0.3990</td>
<td>0.3990</td>
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<tr>
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<td>-0.0001</td>
<td>-0.0001</td>
<td>-0.0001</td>
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</tr>
</tbody>
</table>
Table 7 provides an overview of all quantile regressions. The regressions have been run for all subsets at the 5%, 2.5% and 1% quantile, and all at a 95% confidence level, in order to test the influence of outliers on the overall distribution. The table contains estimates for extreme market conditions. A comparison between all x-variables of quantile regressions those of the OLS-models show only little difference at all and for all data subsets. This means that extreme values have no significant influence on the distribution during the period in question. Even the subsets show little change to those results. What is important, is that several studies have documented a change in correlation of markets over time. Solnik, Boucrelle and Fur (1996) documented the increase in correlations over time and especially during more volatile periods. It is also widely documented that markets experience periods where volatility suddenly increases or decreases (see Sarica and Granger (2005)).

Hence, the VIX shows a great safe haven proposition at a high significance. The proposition as safe haven for the overall stock market holds for the overall data set as well as for all subsets, which include several global crisis events, changing volatility- and market regimes. These results are in line with Hood and Malik, 2013. Also the VVIX has a safe haven proposition, albeit a weak one.
CONCLUSIONS

In this paper, an evaluation of the role of volatility (VIX) and volatility of volatility (VVIX) as potential hedge or safe haven (negative correlation with stocks during extreme stock market declines) in the US stock market (represented by SPX) was undertaken. The findings of this paper is beneficial to both institutional investors – asset managers, pension funds, insurance companies, hedge funds or financial institutions – and academia, as it can provide insights on new diversification, hedging and tail risk hedging strategies. Furthermore, this paper’s findings confirm the safe haven proposition of volatility found in previous studies as Hood and Malik (2013) by utilizing a new methodology (quantile regression). In addition out work as also considering volatility of volatility.

Using daily data from January 1990 to May 2013 (for SPX and VIX) and from August 2006 to May 2013 (for VIX, VVIX and SPX), analysis found that the VIX index serves as a very strong hedge and safe haven for the US stock market. In order to derive this conclusion several regression models were conducted for different data sets to analyze correlation coefficients of each index and time period. The safe haven proposition was tested by utilizing a quantile regression. These models for the same data sets and all subsets for 5%, 2.5% and 1% quantiles were run. These represent the 5%, 2.5% and 1% most extreme outliers. The coefficients for the
quantile models did not vary significantly from the results of the OLS regression models. Hence, extreme values have no influence on the distribution.

It was found that the VVIX is a weak hedge and safe haven instrument for the SPX with lower statistical significance. These propositions hold during changing market environments and different volatility regimes. Furthermore, the results show - according to our definitions - that the VVIX can merely be used as a diversifier of a VIX futures portfolio, as a positive correlation coefficient was found for all subsets.

The correlation of both volatility indices with the overall US-American stock market remains negative at all times and is even stronger in times of crisis events. The overall results suggest that VIX is a superior hedging tool and serves as a better tail risk hedge / safe haven than the VVIX during the sample period. When the correlation matrix was observed for the latest subset, what is visible is an increase in negative correlation of VIX and SPX and also VVIX and SPX. This is a great feature for a safe haven asset, as the safe haven proposition becomes even stronger, when needed most.

These results show that the research’s null-hypothesis: “VIX and VVIX are not affected by (extreme) price movements in equities” holds and is acceptable. The findings are especially valuable for institutional investors and academia as the literature review did not uncover any
other study looking at the relationship of volatility of volatility or using quantile regression in order to test the safe haven proposition of volatility. Utilizing the VIX as a safe haven in combination with VVIX as a diversifier can protect investor portfolios from drawdowns of future financial crises and time-varying co-volatility effects, as common diversification might not suffice. Today, industry practitioners are paying more attention to tail risk than before the bankruptcy of Lehmann Brothers Inc., and discussions of other safe haven assets are controversial and do not always offer clear results. Hence, the results can provide the basis for an alternative strategy and prepare an investor’s portfolio for the next crisis.

The downside of this hedging strategy would be ongoing hedging cost for rolling future contracts. These transaction costs could be significant over the long-run. However, volatility investments could potentially generate profits during normal times as well, if the fund manager, for example shorts the VIX and/or VVIX. Furthermore, as investing in both, VIX and VVIX, requires the use of futures and / or options, a hedging strategy based on these findings would not be open to retail investors as they would lack sophistication and financial means. This model does not consider any tax effects or regulatory restrictions or effects caused by minimum contract size requirements.
Another limitation of this model is the relatively short period of data that is available for the VVIX index. Furthermore, the VVIX is – to date – not as popular amongst practitioners as the VIX already is. The reader should also consider the arbitrary nature of the cutoff dates for the subsets chosen in this work.

The model also does not include any qualitative factors at this point in time and is based on quantitative factors; therefore it only provides limited insights on behavioral aspects.

In addition, due to the methodology used, heteroscedasticity might also limit the results of this work.
ENCOURAGEMENTS FOR SUBSEQUENT RESEARCH

CBOE has announced a whole new family of volatility products in 2012. Amongst them is a Crude Oil ETF\(^{20}\) Volatility securities futures (OV), which started trading on March 26, 2012, and options on CBOE Crude Oil ETF Volatility Index (OVX) started trading on April 10, 2012. Promising source for additional research also seems to be the CBOE Emerging Markets ETF Volatility Index, as well as the CBOE Brazil ETF Volatility Index.\(^{21}\)

All three data sets – and especially the VVIX – show high values of kurtosis. Future research could work on GARCH models in order to account for heteroscedasticity in the time series data. An inclusion of qualitative measures might also improve the informative value of the model. It has been generally accepted that multivariate models are appropriate for transmission mechanisms and correlation dynamics (see Martens and Poon (2001)).

A further extension could be the inclusion of additional, international equity markets or additional asset classes. This would be in order to gain a broader overview of relations between

\(^{20}\)ETF stands for “Exchange Traded Fund”

different asset classes as bonds, commodities or foreign exchange and volatility / volatility of volatility. Karunanayake, Valadkhani, and O’Brien (2010) found diversification amongst smaller international markets to be biggest. Hence future studies could focus on relatively small markets in order to optimize the diversification proposition.

The inclusion of qualitative measures could provide insights into behavioral finance. Interesting might be a focus around market liberalization events in emerging markets similar to Eizaguirre et al.’s (2009) study. Furthermore, the inclusion of liquidity measures such as bid-ask-spreads could add valuable information as liquidity is crucial in times of market crisis.

Future research could consider taking out some data during the peak of each crisis in order to smooth those times of extreme panic. However, the selection process might be arbitrary.

As this model only considers a one-dimensional view of volatility – as both indices focus on implied volatility – future research could evaluate a two-dimensional volatility approach that would include historical volatility.

The subsets chosen in this paper are to some degree arbitrary. Future research could choose more and equally sized subsets.
FIGURES & TABLES

Figures


Figure 2: Term Structure of VVIX; CBOE (2012b), “DOUBLE THE FUN WITH CBOE’s VVIX Index”; 03/14/2013, http://www.cboe.com/micro/VVIX/documents/VVIX-termstructure.pdf; retrieved 7/20/2013, 10:15 AM

Tables

Table 1: Descriptive Statistics VIX, VVIX, and SPX; the author

Table 2: Correlation of VIX, VVIX and SPX until Lehmann Crisis; the author

Table 3: Correlation of VIX, VVIX and SPX after Lehmann Crisis; the author

Table 4: Overview of all correlation matrixes; the author
Table 5: Overview of Descriptive Statistics of Subsets; the author

Table 6: Overview of OLS Regression Models; the author

Table 7: Overview of Quantile Regression Models; the author
REFERENCES


Mosteller, Frederick and John Tukey (1977), „Data Analysis and Regression: A Second Course in Statistics”, Reading, Mass.: Addison-Wesley, p. 266


65


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Pareto, V. (1897), Cours d’Economie Politique, Lausanne, Switzerland


APPENDIX

Program Inclusion

Code for our Quantile Regression Models in SAS 9.2:

```
proc quantreg data=newdata CI=NONE;
MODEL SPX_1 = VIX_1 /quantile=0.01 seed=1916;
RUN;

proc quantreg data=newdata CI=NONE;
MODEL SPX_2 = VIX_2 /quantile=0.01 seed=673;
RUN;

proc quantreg data=newdata CI=NONE;
MODEL SPX_3 = VIX_3 /quantile=0.01 seed=2137;
RUN;

proc quantreg data=newdata CI=NONE;
MODEL SPX_4 = VIX_4 /quantile=0.01 seed=532;
RUN;

proc quantreg data=newdata CI=NONE;
MODEL SPX_5 = VIX_5 /quantile=0.01 seed=1171;
RUN;

proc quantreg data=newdata CI=NONE;
MODEL SPX_4 = VVIX_4 /quantile=0.01 seed=532;
RUN;

proc quantreg data=newdata CI=NONE;
MODEL SPX_5 = VVIX_5 /quantile=0.01 seed=1171;
RUN;

proc quantreg data=newdata CI=NONE;
```
MODEL SPX_allVVIX = VIX_allVVIX /quantile=0.01 seed=1702;
RUN;

proc quantreg data=newdata CI=NONE;
MODEL SPX_allVVIX = VVIX_allVVIX /quantile=0.01 seed=1702;
RUN;
BIOGRAPHICAL SKETCH

Niklas studied chemistry at Ruprecht-Karls University in Heidelberg, Germany, participated in exchange programs with the Higher School of Economics Moscow, Russian Federation and at Polytechnic University Hong Kong, Hong Kong S.A.R. and Graduated from the University of Cooperative Studies Mannheim, Germany with a German Diploma in Business Administration. This thesis is part of Niklas’ degree fulfillment requirements of the international M.B.A. program of the University of North Carolina Wilmington, United States and University of Applied Sciences Bremen, Germany.

Niklas has worked in the past in investment banking, wealth management and consulting for various companies in Germany, the United States, the Netherlands, Switzerland and Hong Kong. He is currently pursuing the CAIA designation as a CAIA-candidate.